

## Hierarchical Modelling for Spatialtemporal Data

Sudipto Banerjee<sup>1</sup> and Andrew O. Finley<sup>2</sup>

<sup>1</sup> Department of Forestry & Department of Geography, Michigan State University, Lansing Michigan, U.S.A.

<sup>2</sup> Biostatistics, School of Public Health, University of Minnesota, Minneapolis, Minnesota, U.S.A.

March 9, 2009

1

Specification:

- Again point-referenced vs. areal unit data
- Continuous time vs. discretized time

⇒ association in space, association in time

For point-referenced data, t continuous, Gaussian

$$Y(\mathbf{s}, t) = \mu(\mathbf{s}, t) + w(\mathbf{s}, t) + \epsilon(\mathbf{s}, t)$$

non-Gaussian data,  $g(EY(\mathbf{s}, t) = \mu(\mathbf{s}, t) + w(\mathbf{s}, t)$

**Don't** treat time as a third coordinate ( $\mathbf{s}, t$ )

$$Cov(Y(\mathbf{s}, t), Y(\mathbf{s}', t')) = C(\mathbf{s} - \mathbf{s}', t - t')$$

2

ENAR 2009 Hierarchical Modeling and Analysis

- **Separable form:**

$$C(\mathbf{s} - \mathbf{s}', t - t') = \sigma^2 \rho_1(\mathbf{s} - \mathbf{s}'; \phi_1) \rho_2(t - t'; \phi_2)$$

- **Nonseparable form:**

- Sum of independent separable processes
- Mixing of separable covariance functions
- Spectral domain approaches

3

ENAR 2009 Hierarchical Modeling and Analysis

- Time discretized,  $Y_t(\mathbf{s}), t = 1, 2, \dots, T$
- Type of data: time series or cross-sectional
- For time series data, exploratory analysis:
  - Arrange into an  $n \times T$  matrix  $Y$  with entries  $Y_t(\mathbf{s}_i)$
  - Center by row averages of  $Y$  yields  $Y_{rows}$
  - Center by column averages of  $Y$  yields  $Y_{cols}$
  - **sample spatial covariance matrix:**  $\frac{1}{T} Y_{rows} Y_{rows}^T$
  - **sample autocorrelation matrix:**  $\frac{1}{n} Y_{cols}^T Y_{cols}$
  - $E$ , residuals matrix after a regression fitting, Empirical orthogonal functions (EOF)

4

ENAR 2009 Hierarchical Modeling and Analysis

- **Modeling:**  $Y_t(\mathbf{s}) = \mu_t(\mathbf{s}) + w_t(\mathbf{s}) + \epsilon_t(\mathbf{s})$ , or perhaps  $g(E(Y_t(\mathbf{s}))) = \mu_t(\mathbf{s}) + w_t(\mathbf{s})$
- For  $\epsilon_t(\mathbf{s})$ , i.i.d.  $N(0, \tau_t^2)$
- For  $w_t(\mathbf{s})$ 
  - $w_t(\mathbf{s}) = \alpha_t + w(\mathbf{s})$
  - $w_t(\mathbf{s})$  independent for each  $t$
  - $w_t(\mathbf{s}) = w_{t-1}(\mathbf{s}) + \eta_t(\mathbf{s})$ , independent spatial process innovations

5

ENAR 2009 Hierarchical Modeling and Analysis

### Dynamic spatiotemporal models

#### Measurement Equation

$$Y(\mathbf{s}, t) = \mu(\mathbf{s}, t) + \epsilon(\mathbf{s}, t); \quad \epsilon(\mathbf{s}, t) \stackrel{ind}{\sim} N(0, \sigma_\epsilon^2).$$

$$\mu(\mathbf{s}, t) = \mathbf{x}(\mathbf{s}, t)' \tilde{\beta}(\mathbf{s}, t).$$

$$\tilde{\beta}(\mathbf{s}, t) = \beta_t + \beta(\mathbf{s}, t)$$

#### Transition Equation

$$\beta_t = \beta_{t-1} + \eta_t, \quad \eta_t \stackrel{ind}{\sim} N_p(\mathbf{0}, \Sigma_\eta)$$

$$\beta(\mathbf{s}, t) = \beta(\mathbf{s}, t-1) + \mathbf{w}(\mathbf{s}, t).$$

6

ENAR 2009 Hierarchical Modeling and Analysis