Hierarchical Modelling for Spatialtemporal Data

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Spatio-temporal Models

Specification:

- Again point-referenced vs. areal unit data
- Continuous time vs. discretized time

 \Rightarrow association in space, association in time For point-referenced data, t continuous, Gaussian $Y(\mathbf{s},t)=\mu(\mathbf{s},t)+w(\mathbf{s},t)+\epsilon(\mathbf{s},t)$

non-Gaussian data, $g(EY(\mathbf{s},t)=\mu(\mathbf{s},t)+w(\mathbf{s},t)$

Don't treat time as a third coordinate (\mathbf{s}, t)

$$Cov(Y(\mathbf{s},t),Y(\mathbf{s}',t')) = C(\mathbf{s}-\mathbf{s}',t-t')$$

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Spatio-temporal Models

• Separable form:

$$C(\mathbf{s} - \mathbf{s}', t - t') = \sigma^2 \rho_1(\mathbf{s} - \mathbf{s}'; \boldsymbol{\phi}_1) \rho_2(t - t'; \boldsymbol{\phi}_2)$$

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- Nonseparable form:
 - Sum of independent separable processes
 - Mixing of separable covariance functions
 - Spectral domain approaches

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Spatio-temporal Models

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 - E, residuals matrix after a regression fitting, Empirical orthogonal functions (EOF)

Spatio-temporal Models

• Modeling: $Y_t(\mathbf{s}) = \mu_t(\mathbf{s}) + w_t(\mathbf{s}) + \epsilon_t(\mathbf{s})$, or perhaps $g(E(Y_t(\mathbf{s})) = \mu_t(\mathbf{s}) + w_t(\mathbf{s})$

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- For $\epsilon_t(\mathbf{s})$, i.i.d. $N(0, \tau_t^2)$
- For $w_t(\mathbf{s})$
 - $w_t(\mathbf{s}) = \alpha_t + w(\mathbf{s})$
 - ullet $w_t(\mathbf{s})$ independent for each t
 - $w_t(\mathbf{s}) = w_{t-1}(\mathbf{s}) + \eta_t(\mathbf{s})$, independent spatial process innovations

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Spatio-temporal Model

Dynamic spatiotemporal models

Measurement Equation

$$\begin{split} Y(\mathbf{s},t) &= \mu(\mathbf{s},t) + \epsilon(\mathbf{s},t); \ \epsilon(\mathbf{s},t) \stackrel{ind}{\sim} N(0,\sigma_{\epsilon}^2). \\ \mu(\mathbf{s},t) &= \mathbf{x}(\mathbf{s},t)' \tilde{\boldsymbol{\beta}}(\mathbf{s},t). \\ \tilde{\boldsymbol{\beta}}(\mathbf{s},t) &= \boldsymbol{\beta}_t + \boldsymbol{\beta}(\mathbf{s},t) \end{split}$$

Transition Equation

$$\begin{split} \boldsymbol{\beta}_t &= \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t, \ \boldsymbol{\eta}_t \overset{ind}{\sim} N_p(\mathbf{0}, \boldsymbol{\Sigma} \boldsymbol{\eta}) \\ \boldsymbol{\beta}(\mathbf{s}, t) &= \boldsymbol{\beta}(\mathbf{s}, t-1) + \mathbf{w}(\mathbf{s}, t). \end{split}$$

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