Hierarchical Modelling for Spatialtemporal Data

Sudipto Banerjee[1] and Andrew O. Finley[2]

[1] Biostatistics, School of Public Health, University of Minnesota, Minneapolis, Minnesota, U.S.A.

[2] Department of Forestry & Department of Geography, Michigan State University, Lansing Michigan, U.S.A.

July 19, 2009

Specification:

- Again point-referenced vs. areal unit data
- Continuous time vs. discretized time

$\Rightarrow$ association in space, association in time
For point-referenced data, t continuous, Gaussian
$Y(\mathbf{s},t) = \mu(\mathbf{s},t) + w(\mathbf{s},t) + \epsilon(\mathbf{s},t)$

non-Gaussian data, $g(EY(\mathbf{s},t) = \mu(\mathbf{s},t) + w(\mathbf{s},t)$

Don't treat time as a third coordinate $(\mathbf{s},t)$

$$Cov(Y(\mathbf{s},t), Y(\mathbf{s}',t')) = C(\mathbf{s}-\mathbf{s}', t-t')$$

- Separable form:

$$C(\mathbf{s}-\mathbf{s}', t-t') = \sigma^2 \rho_1(\mathbf{s}-\mathbf{s}'; \phi_1)\rho_2(t-t'; \phi_2)$$

- Separable form:

$$C(\mathbf{s}-\mathbf{s}', t-t') = \sigma^2 \rho_1(\mathbf{s}-\mathbf{s}'; \phi_1)\rho_2(t-t'; \phi_2)$$

- Nonseparable form:
  - Sum of independent separable processes
  - Mixing of separable covariance functions
  - Spectral domain approaches

- Time discretized, $Y_t(\mathbf{s}), t = 1, 2, ...T$

- Time discretized, $Y_t(\mathbf{s}), t = 1, 2, ...T$
- Type of data: time series or cross-sectional

- Time discretized, $Y_t(\mathbf{s}), t = 1, 2, ...T$
- Type of data: time series or cross-sectional
- For time series data, exploratory analysis:

- Time discretized, $Y_t(\mathbf{s}), t = 1, 2, ...T$
- Type of data: time series or cross-sectional
- For time series data, exploratory analysis:
  - Arrange into an $n \times T$ matrix Y with entries $Y_t(\mathbf{s}_i)$

- Time discretized, $Y_t(\mathbf{s}), t = 1, 2, ...T$
- Type of data: time series or cross-sectional
- For time series data, exploratory analysis:
  - Arrange into an $n \times T$ matrix Y with entries $Y_t(\mathbf{s}_i)$
  - Center by row averages of Y yields Y_{rows}

- Time discretized, $Y_t(\mathbf{s}), t = 1, 2, ...T$
- Type of data: time series or cross-sectional
- For time series data, exploratory analysis:
  - Arrange into an $n \times T$ matrix Y with entries $Y_t(\mathbf{s}_i)$
  - Center by row averages of Y yields Y_{rows}
  - Center by column averages of Y yields Y_{cols}

- Time discretized, $Y_t(\mathbf{s}), t = 1, 2, ...T$
- Type of data: time series or cross-sectional
- For time series data, exploratory analysis:
  - Arrange into an $n \times T$ matrix Y with entries $Y_t(\mathbf{s}_i)$
  - Center by row averages of Y yields Y_{rows}
  - Center by column averages of Y yields Y_{cols}
  - sample spatial covariance matrix: $\frac{1}{T} Y_{rows} Y_{rows}^T$

- Time discretized, $Y_t(\mathbf{s}), t = 1, 2, ...T$
- Type of data: time series or cross-sectional
- For time series data, exploratory analysis:
  - Arrange into an $n \times T$ matrix Y with entries $Y_t(\mathbf{s}_i)$
  - Center by row averages of Y yields Y_{rows}
  - Center by column averages of Y yields Y_{cols}
  - sample spatial covariance matrix: $\frac{1}{T} Y_{rows} Y_{rows}^T$
  - sample autocorrelation matrix: $\frac{1}{n} Y_{cols}^T Y_{cols}$

- Time discretized, $Y_t(\mathbf{s}), t = 1, 2, ...T$
- Type of data: time series or cross-sectional
- For time series data, exploratory analysis:
  - Arrange into an $n \times T$ matrix Y with entries $Y_t(\mathbf{s}_i)$
  - Center by row averages of Y yields Y_{rows}
  - Center by column averages of Y yields Y_{cols}
  - sample spatial covariance matrix: $\frac{1}{T} Y_{rows} Y_{rows}^T$
  - sample autocorrelation matrix: $\frac{1}{n} Y_{cols}^T Y_{cols}$
  - E, residuals matrix after a regression fitting, Empirical orthogonal functions (EOF)

- Modeling: $Y_t(\mathbf{s}) = \mu_t(\mathbf{s}) + w_t(\mathbf{s}) + \epsilon_t(\mathbf{s})$, or perhaps $g(E(Y_t(\mathbf{s})) = \mu_t(\mathbf{s}) + w_t(\mathbf{s})$

- Modeling: $Y_t(\mathbf{s}) = \mu_t(\mathbf{s}) + w_t(\mathbf{s}) + \epsilon_t(\mathbf{s})$, or perhaps $g(E(Y_t(\mathbf{s})) = \mu_t(\mathbf{s}) + w_t(\mathbf{s})$
- For $\epsilon_t(\mathbf{s})$, i.i.d. $N(0, \tau_t^2)$

- Modeling: $Y_t(\mathbf{s}) = \mu_t(\mathbf{s}) + w_t(\mathbf{s}) + \epsilon_t(\mathbf{s})$, or perhaps $g(E(Y_t(\mathbf{s})) = \mu_t(\mathbf{s}) + w_t(\mathbf{s})$
- For $\epsilon_t(\mathbf{s})$, i.i.d. $N(0, \tau_t^2)$
- For $w_t(\mathbf{s})$
  - $w_t(\mathbf{s}) = \alpha_t + w(\mathbf{s})$
  - $w_t(\mathbf{s})$ independent for each t
  - $w_t(\mathbf{s}) = w_{t-1}(\mathbf{s}) + \eta_t(\mathbf{s})$, independent spatial process innovations

Dynamic spatiotemporal models

Measurement Equation

$$Y(\mathbf{s},t) = \mu(\mathbf{s},t) + \epsilon(\mathbf{s},t); \quad \epsilon(\mathbf{s},t) \overset{ind}{\sim} N(0,\sigma_\epsilon^2).$$
$$\mu(\mathbf{s},t) = \mathbf{x}(\mathbf{s},t)'\tilde{\boldsymbol{\beta}}(\mathbf{s},t).$$
$$\tilde{\boldsymbol{\beta}}(\mathbf{s},t) = \boldsymbol{\beta}_t + \boldsymbol{\beta}(\mathbf{s},t)$$

Transition Equation

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t, \ \boldsymbol{\eta}_t \overset{ind}{\sim} N_p(\mathbf{0}, \Sigma_{\boldsymbol{\eta}})$$
$$\boldsymbol{\beta}(\mathbf{s},t) = \boldsymbol{\beta}(\mathbf{s},t-1) + \mathbf{w}(\mathbf{s},t).$$