

# Illustration of Adaptive MCMC

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## 1 Overview

The `adaptMetropGibb` function in `spBayes` generates MCMC samples for a continuous random vector using an adaptive Metropolis within Gibbs algorithm (see Roberts and Rosenthal, 2006; Rosenthal, 2007). We illustrate this function by fitting a simple univariate spatial regression model to some synthetic data.

In the code block below, we generate some synthetic data then set our priors. We use a logit transformation to project  $\phi$  to the real-line and log transform  $\sigma^2$  and  $\tau^2$ . These transformations necessitate computing their respective Jacobians in the log target density.

```
> set.seed(1)
> n <- 50
> x <- runif(n, 0, 100)
> y <- runif(n, 0, 100)
> D <- as.matrix(dist(cbind(x, y)))
> phi <- 3/50
> sigmasq <- 50
> tausq <- 20
> mu <- 150
> s <- (sigmasq * exp(-phi * D))
> w <- mvrnorm(1, rep(0, n), s)
> Y <- mvrnorm(1, rep(mu, n) + w, tausq * diag(n))
> X <- as.matrix(rep(1, length(Y)))

> a.sig <- 2
> b.sig <- 100
> a.tau <- 2
> b.tau <- 100
> a.phi <- 3/500
> b.phi <- 3/3
> logit <- function(theta, a, b) {
+   log((theta - a)/(b - theta))
+ }
> logit.inv <- function(z, a, b) {
```

```

+   b - (b - a)/(1 + exp(z))
+ }
> target <- function(theta) {
+   mu.cand <- theta[1]
+   sigmasq.cand <- exp(theta[2])
+   tausq.cand <- exp(theta[3])
+   phi.cand <- logit.inv(theta[4], a.phi, b.phi)
+   Sigma <- sigmasq.cand * exp(-phi.cand * D) +
+     tausq.cand * diag(n)
+   SigmaInv <- chol2inv(chol(Sigma))
+   logDetSigma <- determinant(Sigma, log = TRUE)$modulus[1]
+   out <- (-(a.sig + 1) * log(sigmasq.cand) - b.sig/sigmasq.cand -
+     (a.tau + 1) * log(tausq.cand) - b.tau/tausq.cand +
+     log(sigmasq.cand) + log(tausq.cand) + log(phi.cand -
+     a.phi) + log(b.phi - phi.cand) - 0.5 * logDetSigma -
+     0.5 * (t(Y - X %>% mu.cand) %>% SigmaInv %>%
+     (Y - X %>% mu.cand)))
+   return(out)
+ }
> inits <- c(0, log(1000), log(1000), logit(3/10, a.phi,
+   b.phi))
> metrop.out <- adaptMetropGibbs(ltd = target, starting = inits,
+   batch = 500, batch.length = 25, report = 100)

```

-----  
 Sampling  
 -----

Batch: 100 of 500  
 Metropolis batch acceptance rate:  
 0.560      0.360      0.440      0.480  
 -----

Batch: 200 of 500  
 Metropolis batch acceptance rate:  
 0.360      0.560      0.520      0.520  
 -----

Batch: 300 of 500  
 Metropolis batch acceptance rate:  
 0.560      0.360      0.400      0.560  
 -----

Batch: 400 of 500  
 Metropolis batch acceptance rate:  
 0.320      0.280      0.360      0.440  
 -----

```
> summary(mcmc(metrop.out$acceptance))
```

Iterations = 1:500

```
Thinning interval = 1
Number of chains = 1
Sample size per chain = 500
```

1. Empirical mean and standard deviation for each variable,  
plus standard error of the mean:

	Mean	SD	Naive SE	Time-series SE
[1,]	0.5576	0.18971	0.008484	0.020944
[2,]	0.4418	0.10628	0.004753	0.006759
[3,]	0.4514	0.09931	0.004441	NA
[4,]	0.4806	0.12839	0.005742	0.008729

2. Quantiles for each variable:

	2.5%	25%	50%	75%	97.5%
var1	0.240	0.40	0.56	0.68	0.960
var2	0.200	0.36	0.44	0.52	0.640
var3	0.259	0.40	0.44	0.52	0.640
var4	0.240	0.40	0.48	0.56	0.741

```
> plot(mcmc(metrop.out$acceptance))
```

```
> p.samples <- metrop.out$p.samples
> p.samples[, 2] <- exp(metrop.out$p.samples[, 2])
> p.samples[, 3] <- exp(metrop.out$p.samples[, 3])
> p.samples[, 4] <- 3/logit.inv(metrop.out$p.samples[,
+ 4], a.phi, b.phi)
> colnames(p.samples) <- c("mu", "sigma.sq", "tau.sq",
+ "effective range")
> plot(mcmc(p.samples), smooth = FALSE, density = FALSE)
```

```
> n.samples <- nrow(p.samples)
> burn.in <- as.integer(0.25 * n.samples)
> p.samples <- mcmc(p.samples[burn.in:n.samples, ])
> summary(p.samples)
```

```
Iterations = 1:9376
Thinning interval = 1
Number of chains = 1
Sample size per chain = 9376
```

1. Empirical mean and standard deviation for each variable,  
plus standard error of the mean:

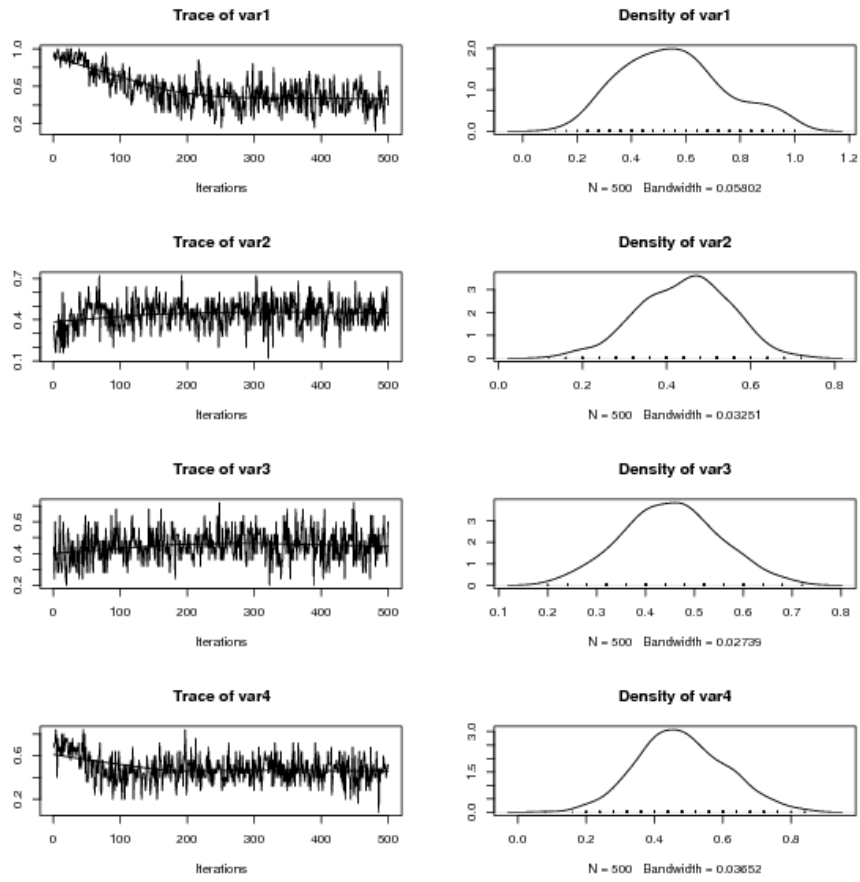


Figure 1: Metropolis acceptance trace plots.

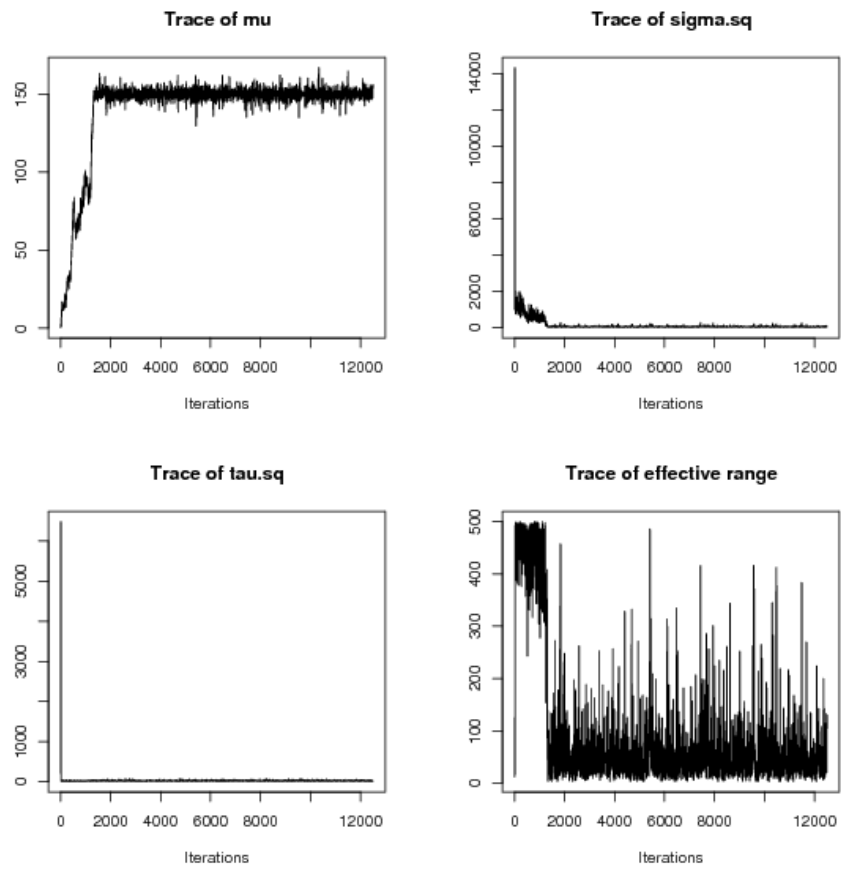


Figure 2: MCMC trace plots.

	Mean	SD	Naive SE	Time-series SE
mu	150.06	2.942	0.03038	0.08333
sigma.sq	54.77	26.749	0.27625	1.21901
tau.sq	26.50	9.924	0.10249	0.33701
effective range	53.46	48.857	0.50457	2.06771

2. Quantiles for each variable:

	2.5%	25%	50%	75%	97.5%
mu	143.643	148.62	150.12	151.63	155.92
sigma.sq	23.535	37.78	49.05	64.04	125.18
tau.sq	12.401	19.43	24.75	31.37	50.54
effective range	6.187	25.33	40.01	63.77	190.43

> plot(p.samples, density = FALSE)

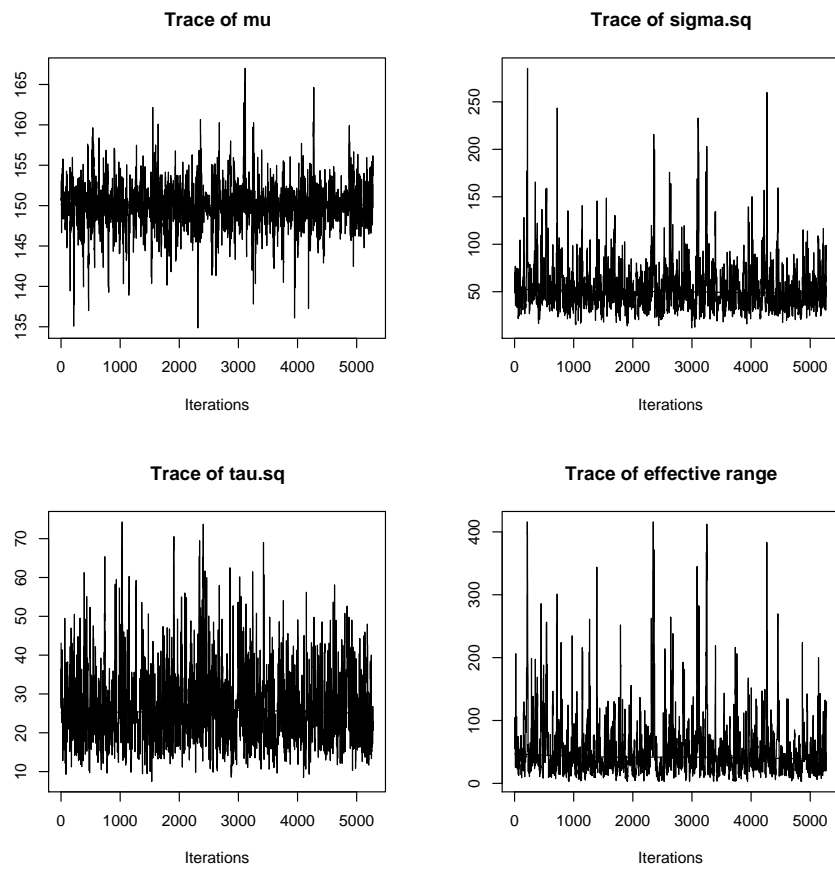


Figure 3: MCMC trace plots post burn-in.

## 2 References

- Roberts G.O. and Rosenthal J.S. (2006). Examples of Adaptive MCMC. <http://probability.ca/jeff/ftpd/adaptex.pdf> Preprint.
- Rosenthal J.S. (2007). AMCMC: An R interface for adaptive MCMC. *Computational Statistics and Data Analysis*, 51:5467-5470.