

Illustration of Adaptive MCMC

Andrew O. Finley and Sudipto Banerjee

June 9, 2013

1 Overview

The `adaptMetropGibb` function in `spBayes` generates MCMC samples for a continuous random vector using an adaptive Metropolis within Gibbs algorithm (see Roberts and Rosenthal, 2006; Rosenthal, 2007). We illustrate this function by fitting a simple univariate spatial regression model to some synthetic data.

In the code block below, we generate some synthetic data then set our priors. We use a logit transformation to project ϕ to the real-line and log transform σ^2 and τ^2 . These transformations necessitate computing their respective Jacobians in the log target density.

```
> set.seed(1)
> n <- 50
> x <- runif(n, 0, 100)
> y <- runif(n, 0, 100)
> D <- as.matrix(dist(cbind(x, y)))
> phi <- 3/50
> sigmasq <- 50
> tausq <- 20
> mu <- 150
> s <- (sigmasq * exp(-phi * D))
> w <- mvrnorm(1, rep(0, n), s)
> Y <- mvrnorm(1, rep(mu, n) + w, tausq * diag(n))
> X <- as.matrix(rep(1, length(Y)))

> a.sig <- 2
> b.sig <- 100
> a.tau <- 2
> b.tau <- 100
> a.phi <- 3/500
> b.phi <- 3/3
> logit <- function(theta, a, b) {
+   log((theta - a)/(b - theta))
+ }
> logit.inv <- function(z, a, b) {
```

```

+   b - (b - a)/(1 + exp(z))
+ }
> target <- function(theta) {
+   mu.cand <- theta[1]
+   sigmasq.cand <- exp(theta[2])
+   tausq.cand <- exp(theta[3])
+   phi.cand <- logit.inv(theta[4], a.phi, b.phi)
+   Sigma <- sigmasq.cand * exp(-phi.cand * D) +
+     tausq.cand * diag(n)
+   SigmaInv <- chol2inv(chol(Sigma))
+   logDetSigma <- determinant(Sigma, log = TRUE)$modulus[1]
+   out <- (-(a.sig + 1) * log(sigmasq.cand) - b.sig/sigmasq.cand -
+     (a.tau + 1) * log(tausq.cand) - b.tau/tausq.cand +
+     log(sigmasq.cand) + log(tausq.cand) + log(phi.cand -
+     a.phi) + log(b.phi - phi.cand) - 0.5 * logDetSigma -
+     0.5 * (t(Y - X %>% mu.cand) %>% SigmaInv %>%
+     (Y - X %>% mu.cand)))
+   return(out)
+ }
> inits <- c(0, log(1000), log(1000), logit(3/10, a.phi,
+   b.phi))
> metrop.out <- adaptMetropGibbs(ltd = target, starting = inits,
+   batch = 500, batch.length = 25, report = 100)

```

 Sampling

Batch: 100 of 500
 Metropolis batch acceptance rate:
 0.560 0.440 0.600 0.520

Batch: 200 of 500
 Metropolis batch acceptance rate:
 0.320 0.560 0.520 0.600

Batch: 300 of 500
 Metropolis batch acceptance rate:
 0.520 0.480 0.360 0.280

Batch: 400 of 500
 Metropolis batch acceptance rate:
 0.360 0.360 0.400 0.560

```
> summary(mcmc(metrop.out$acceptance))
```

Iterations = 1:500

```

Thinning interval = 1
Number of chains = 1
Sample size per chain = 500

```

1. Empirical mean and standard deviation for each variable, plus standard error of the mean:

	Mean	SD	Naive SE	Time-series SE
[1,]	0.5378	0.17920	0.008014	0.067818
[2,]	0.4486	0.11119	0.004973	0.007193
[3,]	0.4495	0.09611	0.004298	0.004298
[4,]	0.4758	0.12889	0.005764	0.019530

2. Quantiles for each variable:

	2.5%	25%	50%	75%	97.5%
var1	0.24	0.40	0.52	0.64	0.960
var2	0.20	0.36	0.44	0.52	0.661
var3	0.28	0.40	0.44	0.52	0.640
var4	0.24	0.40	0.48	0.56	0.720

```

> plot(mcmc(metrop.out$acceptance))

> p.samples <- metrop.out$p.theta.samples
> p.samples[, 2] <- exp(metrop.out$p.theta.samples[,
+ 2])
> p.samples[, 3] <- exp(metrop.out$p.theta.samples[,
+ 3])
> p.samples[, 4] <- 3/logit.inv(metrop.out$p.theta.samples[,
+ 4], a.phi, b.phi)
> colnames(p.samples) <- c("mu", "sigma.sq", "tau.sq",
+ "effective range")
> plot(mcmc(p.samples), smooth = FALSE, density = FALSE)

> n.samples <- nrow(p.samples)
> burn.in <- as.integer(0.25 * n.samples)
> p.samples <- mcmc(p.samples[burn.in:n.samples, ])
> summary(p.samples)

Iterations = 1:9376
Thinning interval = 1
Number of chains = 1
Sample size per chain = 9376

```

1. Empirical mean and standard deviation for each variable,

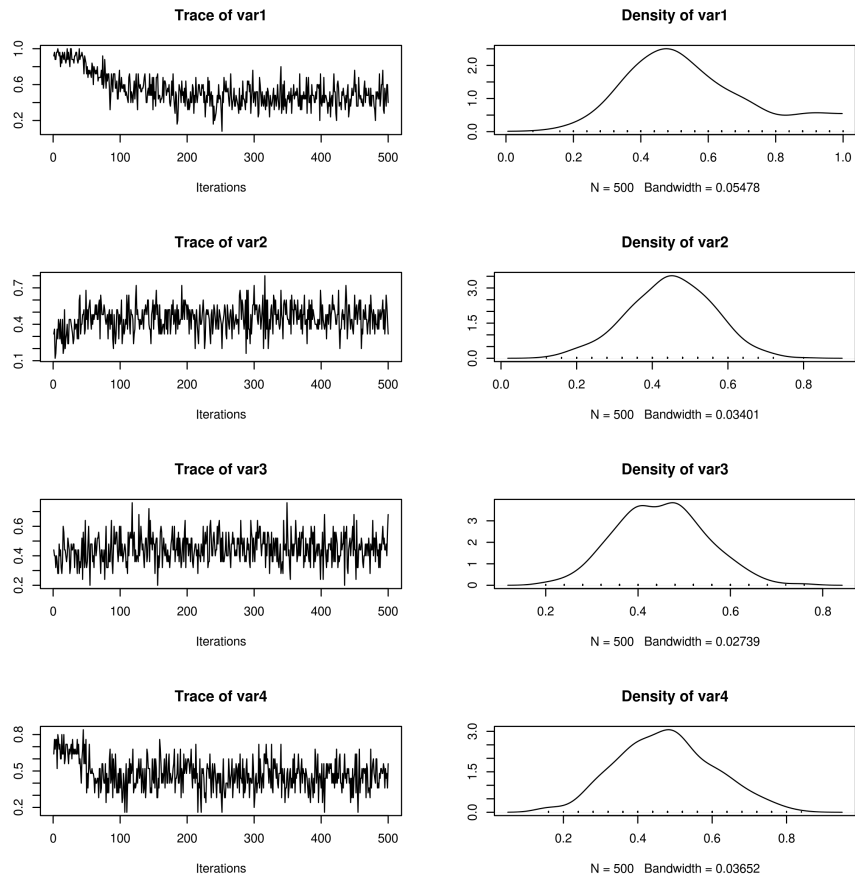


Figure 1: Metropolis acceptance trace plots.

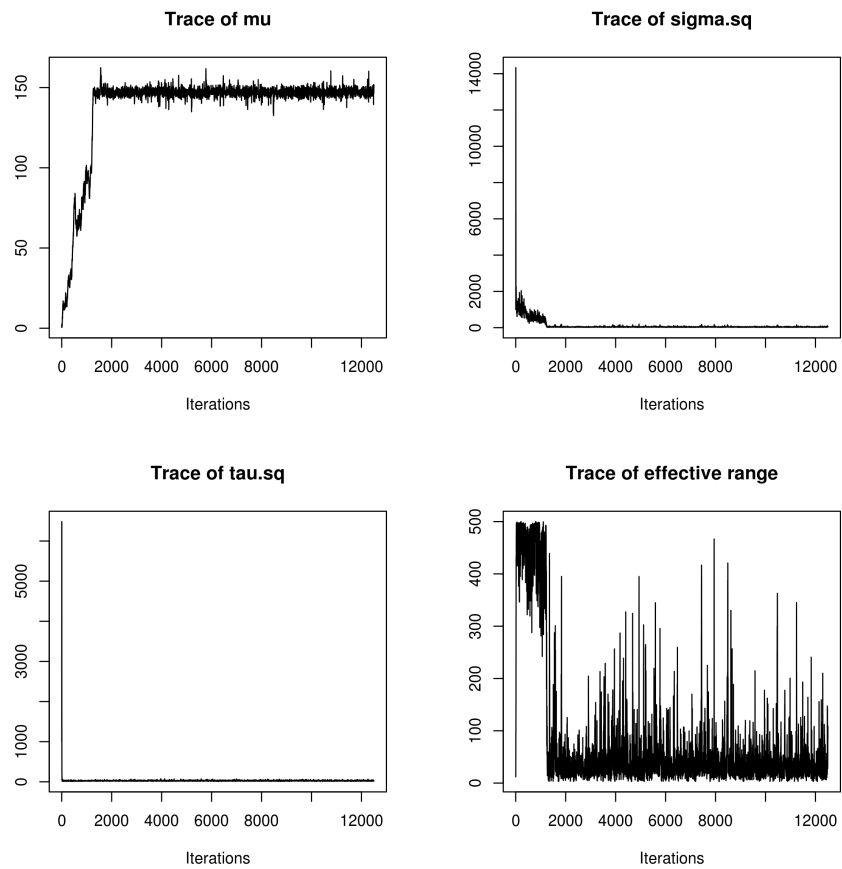


Figure 2: MCMC trace plots.

plus standard error of the mean:

	Mean	SD	Naive SE	Time-series SE
mu	147.18	2.249	0.02322	0.05978
sigma.sq	41.01	17.395	0.17964	0.53887
tau.sq	26.49	9.407	0.09714	0.26527
effective range	40.72	40.819	0.42155	1.57664

2. Quantiles for each variable:

	2.5%	25%	50%	75%	97.5%
mu	142.633	146.07	147.21	148.38	151.57
sigma.sq	18.768	29.73	37.47	48.18	83.00
tau.sq	12.814	19.80	24.58	31.76	49.77
effective range	5.345	19.36	29.54	45.64	149.32

> plot(p.samples, density = FALSE)

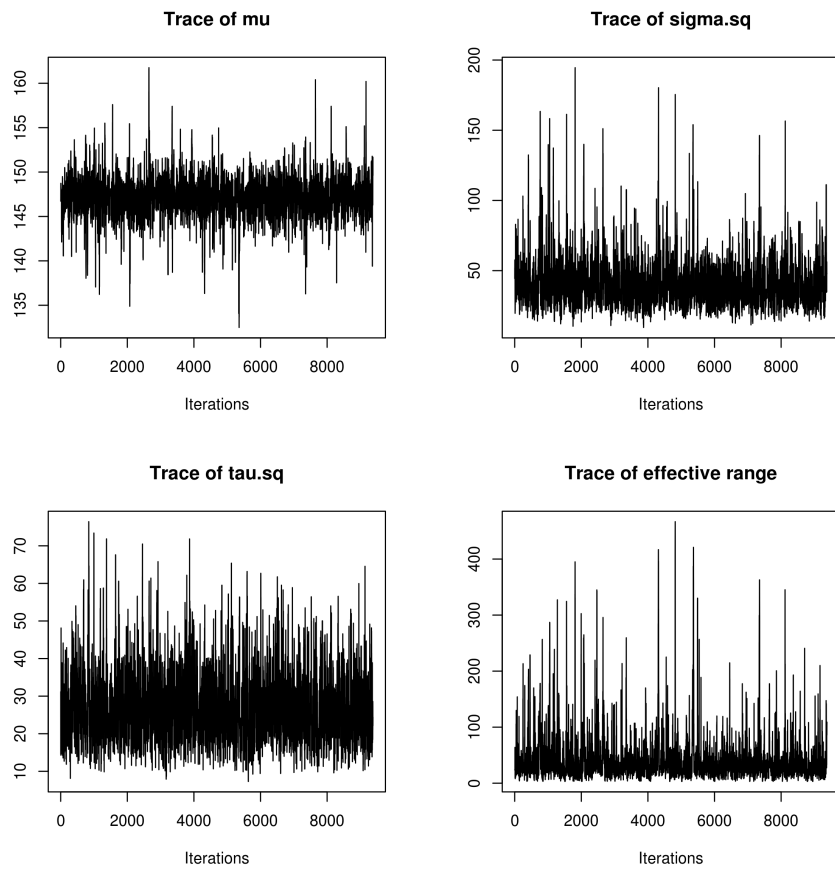


Figure 3: MCMC trace plots post burn-in.

2 References

- Roberts G.O. and Rosenthal J.S. (2006). Examples of Adaptive MCMC. <http://probability.ca/jeff/ftpd/adaptex.pdf> Preprint.
- Rosenthal J.S. (2007). AMCMC: An R interface for adaptive MCMC. *Computational Statistics and Data Analysis*, 51:5467-5470.