Hierarchical Modeling for Spatio-temporal Data

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Specification:

- Again point-referenced vs. areal unit data
- Continuous time vs. discretized time

$\Rightarrow$ association in space, association in time
For point-referenced data, t continuous, Gaussian
$y(\mathbf{s},t) = \mu(\mathbf{s},t) + w(\mathbf{s},t) + \epsilon(\mathbf{s},t)$

non-Gaussian data, $g(E[y(\mathbf{s},t)]) = \mu(\mathbf{s},t) + w(\mathbf{s},t)$

Don't treat time as a third coordinate $(\mathbf{s},t)$

$$Cov(y(\mathbf{s},t), y(\mathbf{s}',t'))) = C(\mathbf{s}-\mathbf{s}', t-t')$$

- Separable form:

$$C(\mathbf{s}-\mathbf{s}', t-t') = \sigma^2\rho_1(\mathbf{s}-\mathbf{s}';\phi_1)\rho_2(t-t';\phi_2)$$

- Nonseparable form:
  - Sum of independent separable processes
  - Mixing of separable covariance functions
  - Spectral domain approaches

- Time discretized, $y_t(\mathbf{s}), t = 1, 2, ...T$
- Type of data: time series or cross-sectional

- Modeling: $y_t(\mathbf{s}) = \mu_t(\mathbf{s}) + w_t(\mathbf{s}) + \epsilon_t(\mathbf{s})$, or perhaps $g(E[y_t(\mathbf{s})]) = \mu_t(\mathbf{s}) + w_t(\mathbf{s})$
- For $\epsilon_t(\mathbf{s})$, i.i.d. $N(0, \tau_t^2)$
- For $w_t(\mathbf{s})$
  - $w_t(\mathbf{s}) = \alpha_t + w(\mathbf{s})$
  - $w_t(\mathbf{s})$ independent for each t
  - $w_t(\mathbf{s}) = w_{t-1}(\mathbf{s}) + \eta_t(\mathbf{s})$, independent spatial process innovations

Dynamic spatiotemporal models

Measurement Equation

$$y(\mathbf{s},t) = \mu(\mathbf{s},t) + \epsilon(\mathbf{s},t);\ \ \epsilon(\mathbf{s},t) \overset{ind}{\sim} N(0,\sigma_\epsilon^2).$$
$$\mu(\mathbf{s},t) = \mathbf{x}(\mathbf{s},t)'\tilde{\boldsymbol{\beta}}(\mathbf{s},t).$$
$$\tilde{\boldsymbol{\beta}}(\mathbf{s},t) = \boldsymbol{\beta}_t + \boldsymbol{\beta}(\mathbf{s},t)$$

Transition Equation

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t,\ \boldsymbol{\eta}_t \overset{ind}{\sim} N_p(\mathbf{0},\Sigma_{\boldsymbol{\eta}})$$
$$\boldsymbol{\beta}(\mathbf{s},t) = \boldsymbol{\beta}(\mathbf{s},t-1) + \mathbf{w}(\mathbf{s},t).$$